SUPPORT REACTIONS

Types of supports

1. Simple support

A simple support is represented as shown in the figure.

A simple support can offer reaction in a direction \( \perp \) to the beam.

2. Roller Support

A roller support is represented as shown in fig.

Roller support offer reaction \( \perp \) to the beam.
3. Hinged (Pinned) Support

Reaction by a hinged support can be inclined to the beam.

In Figure RA is the reaction by the support, RAH is the horizontal component of reaction, RAV is the vertical component of reaction.

4. Support by a plane

Reaction by the plane is always perpendicular to the plane supporting the body.
5. Fixed Support

It is represented as shown below.

\[ \text{Diagram of a fixed support.} \]

Reaction by a fixed support can be inclined to the plane.

\[ \text{Diagram showing reaction forces.} \]

**TYPES OF LOADS**

1) **Point Load**

These are loads acting on a point. Point load is represented as shown in figure.

\[ \text{Diagram of a point load.} \]

2) **Uniformly Distributed Loads (udl)**

These are the loads acting over a portion of the beam. It is represented as in figure.

\[ \text{Diagram of a uniformly distributed load.} \]

\[ \Rightarrow \text{for converting udl into a point load.} \]

**Step 1:** Magnitude of corresponding point load.

\[ \text{ie, } 10 \times 3 = 30 \text{ N} \]
Step 2: Distance of load from point A

\[ 1 + \frac{3}{2} = 2.5 \text{ m} \]

This is the corresponding point load.

A beam of length 8 m is loaded as shown in fig.
Determine the reactions on both ends.

A. FBD

\[ \Sigma F_y = 0 \]
\[ R_A - 20 - 30 + R_B = 0 \]
\[ R_A + R_B = 50 \quad (1) \]

Moment about point A = 0

\[ \Sigma m_A = 0 \]
\[ (R_A \times 0) - (20 \times 3) - (30 \times 6) \quad R_B \times 8 = 0. \quad (2) \]
\[ R_B = \frac{240}{8} = 30 \text{ N} \]
\[ R_A = 20 \text{ N} \]
A beam is hinged at one end and simply supported on the other end. Determine the reactions on both ends.

\[ \Sigma F_x = 0. \]
\[ R_{AH} \cos 30^\circ = 0. \]
\[ R_{AH} = 25.98 \text{ N}. \]

\[ \Sigma F_y = 0. \]
\[ R_{AV} - 100 - 30 \sin 30^\circ + R_B = 0 \]
\[ R_{AV} + R_B = 115 \quad (2) \]

\[ \Sigma M_A = 0. \]
\[ -(100 \times 2.5) - (30 \times 6 \times \sin 30^\circ) + R_B \times 8 = 0 \]
\[ R_B = 42.5 \text{ N}. \]
\[ R_{AV} = 72 \text{ N}. \]
\[ R_A = \sqrt{R_{AV}^2 + R_{AH}^2} = \sqrt{72.5^2 + 25.98^2.} \]
\[ = 89.7 \text{ N}. \]
\[ \theta = \tan^{-1} \left( \frac{RA_1}{RA_H} \right) = \tan^{-1} \left( \frac{72.5}{25.98} \right) \]

\[ = 70.28^\circ \]

? A beam of span 7 m is loaded as shown in fig.
Determine the reactions on both ends.

A FBD

\[ \Sigma F_y = 0 \]

\[ RA - 35 - 15 + RB = 0 \]

\[ RA + RB = 50 \quad (1) \]

\[ M_A = 0 \]

\[ (RA \times 0) - (35 \times 3.5) - (15 \times 5) + RB \times 7 = 0 \]

\[ RB = 98.72 \text{ N} \]

\[ RA = 21.8 \text{ N} \]
A beam of span 6m is supported by two rollers resting on inclined planes. Determine the reactions on both ends.

\[ \Sigma f_x = 0 \]

\[ R_{AW} \cos 50^\circ - R_B \cos 60^\circ + 0.15 \cos 20^\circ = 0 \]

\[ R_A = \frac{0.5 R_B - 14.09}{0.64} \quad (1) \]

\[ \Sigma f_y = 0 \]

\[ 0.64 R_A - 0.5 R_B = -14.09 \quad (1) \]

\[ R_A \sin 50^\circ - 10 - 10 \sin 20^\circ + R_B \sin 60^\circ = 0 \]

\[ 0.77 R_A + 0.87 R_B = 25.13 \quad (2) \]

\[ 0.77 \left( \frac{0.5 R_B - 14.09}{0.64} \right) + 0.87 R_B = 25.13 \]

\[ 1.47 R_B = 42.08 \]

\[ R_B = 28.68 \text{ kN} \]

\[ R_A = 0.88 \text{ kN} \]
A beam of span 7m is loaded as shown in Figure. Determine the reactions on both ends.

\[\sum F_x = 0\]

\[R_{AH} + 10\cos 40^\circ = 0\]

\[R_{AH} = -7.66\,\text{kN}\]

\[\sum F_y = 0\]

\[R_{AV} - 10\sin 40^\circ - 15 - 15 + R_B = 0\]

\[R_{AV} + R_B = 36.48\quad (\text{W})\]

\[\sum M_A = 0\]

\[-(10 \times 2 \times \sin 40^\circ) - (15 \times 3.75) - (15 \times 1) + R_B \times 7 = 0\]

\[R_B = 12.01\,\text{kN}\]

\[R_{AV} = 24.42\,\text{kN}\]

\[R_A = \sqrt{R_{AV}^2 + R_{AH}^2} = \sqrt{24.42^2 + (-7.66)^2}\]

\[= 25.6\,\text{kN}\]

\[\theta = \tan^{-1}\left(\frac{R_{AV}}{R_{AH}}\right) = -72.58^\circ\]
1) A beam 6m long is loaded as shown in figure. Calculate the reactions at A & B.

![Beam figure with loads and reactions]

2) Determine the reactions at the supports.

![Beam figure with loads and reactions]

3) Determine the reactions at the supports.

![Beam figure with loads and reactions]

4) Determine the reactions at A & B.

![Beam figure with loads and reactions]

1) \( \text{FBD.} \)

\[ \sum F_y = 0. \]

\[ R_A - 10 + R_B = 0. \]

\[ R_A + R_B = 10 \quad (1) \]
\[ \Sigma M_A = 0. \]
\[- (10 \times 2) - 8 + R_B \cdot 6 = 0 \]
\[ R_B = +4.67 \text{kN} \]
\[ R_A = +4.67 \text{kN} - 5.33 \text{kN}. \]

2) A FBD

\[ \Sigma F_x = 0 \]
\[- R_B \cos 60^\circ = 0 \]
\[ R_{AH} + 20 \cos 65^\circ - 25 \cos 55^\circ - R_B \cos 60^\circ = 0. \]
\[ R_{AH} - 0.8 R_B = +8.84 \text{kN}. \] (1)

\[ \Sigma F_y = 0 \]
\[ R_{AV} - 20 \sin 65^\circ - 85 - 25 \sin 55^\circ + R_B \sin 60^\circ = 0. \]
\[ R_{AV} + 0.89 R_B = +38.6. \] (2)

\[ \Sigma M_A = 0 \]
\[- (20 \sin 65 \times 1) - (35 \times 2.5) - (25 \sin 55 \times 3.5) + (R_B \sin 60 \times 85) = 0 \]
\[ R_B = 36.14 \text{kN}. \]
\[ R_{AV} = +45.49 \text{kN}. \]
\[ R_{AH} = 28.68 \text{kN}. \]
3) A \[ FBD \]

\[ R_A = 44.46 \]
\[ \theta = \tan^{-1}\left(\frac{R_{AV}}{R_{AH}}\right) = 49.92^\circ \]

\[ \sum F_y = 0 \]
\[ R_A - 10 - 300 + R_B = 0 \]
\[ R_A + R_B = 1200 \quad (1) \]

\[ \sum M_A = 0 \]
\[ -10 \cdot (10 \times 5) - (3 \times 6.5) + (R_B \times 8) = 0 \]
\[ R_B = 9.94 \text{ kN} \]
\[ R_A = 3.06 \text{ kN} \]

4) A \[ FBD \]

\[ \sum F_x = 0 \]
\[ R_{AH} + 4 \cos 45^\circ + 8 = 0 \]
\[ R_{AH} = -10.82 \text{ kN} \]
\[ \sum F_y = 0 \]
\[ R_A - 5 + 4 \sin 45^\circ + R_B = 0 \]
\[ R_A + R_B = 2.17 \quad (2) \]
\[ \sum M_A = 0 \]
\[ - (5 \times 2) + (4 \sin 45^\circ \times 8) - (8 \times 2) \times (R_B \times 8) = 0 \]
\[ R_B = 1.84 \text{ kN} \]
\[ R_A = 0.32 \text{ kN} \]
\[ R_A = 10.82 \text{ kN} \]
\[ \theta = \tan^{-1} \left( \frac{R_A}{R_B} \right) = -1.8^\circ \]

**TYPES OF BEAMS**

1. Simply Supported Beam
2. Cantilever Beam
3. Overhanging Beam
4. Simply Supported Beam is given support on both ends of the beam.

2) A Cantilever Beam is one which is fixed at one end and free on the other end.
3) An overhanging beam extends beyond the support. The portion of beam that extends beyond the support is called an overhanging section. Overhanging sections can be on one side of the beam or on both sides.

A cantilever beam of span 8 m is loaded as shown in Fig. Determine the reactions on the support.

\[ \Sigma F_x = 0 \]
\[ R_H = 0 \]
\[ \Sigma F_y = 0 \]
\[ R_V - 16 - 10 = 0 \]
\[ R_V = 26 \text{ N} \]

Assume moment acting on the support is in an anticlockwise direction.
\[ \Sigma M = 0 \]
\[ m - (16 \times 4) - (10 \times 8) = 0 \]
\[ m = 80 + 6 \phi = \frac{144}{\phi} \text{ N.m} \]

A cantilever is supported as shown in figure. Determine the reaction on the support.

![FBD Diagram](image)

\[ \Sigma F_x = 0 \]
\[ R_H - 30 \cos 20^\circ = 0 \]
\[ R_H = 28.19 \text{ N} \]

\[ \Sigma F_y = 0 \]
\[ R_V - 30 \sin 20^\circ - 15 = 0 \]
\[ R_V = 25.26 \text{ N} \]

\[ \Sigma M = 0 \]
\[ m - (30 \sin 20^\circ \times 4) - (15 \times 5.5) = 0 \]
\[ m = 123.54 \text{ N.m} \]

\[ R = 39.85 \text{ N.m} \quad \theta = 41.86^\circ \]
An overhanging beam of length 9 m is shown in fig. Determine the reactions.

A. FBD

\[ \sum F_y = 0, \]
\[ R_A - 45 + R_B - 10 = 0, \]
\[ R_A + R_B = 55 \quad \text{(1)} \]
\[ \sum M = 0, \]
\[ -(45 \times 4.5) + (6R_B) - (10 \times 9) = 0. \]

\[ R_B = \frac{292.5}{6} = 48.75 \text{ N} \]
\[ R_A = 6.25 \text{ N} \]
FORCES IN SPACE

Let $\mathbf{F}_1$ is a force defined by $F_1 i + F_2 j + F_3 k$ and let $\mathbf{F}_2$ is another force represented by $P_1 i + P_2 j + P_3 k$.

Then $\mathbf{F} \cdot \mathbf{F}_2 = (F_1 || P_1) \cos \theta$, where $\theta$ is the angle between these two vectors.

$\mathbf{F} \cdot \mathbf{F}_2$ is given by,

$$ (F_1 P_1) + (F_2 P_2) + (F_3 P_3) $$

$\mathbf{F} \times \mathbf{F}_2$ is given by,

$$ \begin{vmatrix} i & j & k \\ F_1 & F_2 & F_3 \\ P_1 & P_2 & P_3 \end{vmatrix} $$

Dot product will give a scalar whereas cross product will produce a vector as its output.

? If two vectors are represented by

$\mathbf{A} = 3 \mathbf{i} + 4 \mathbf{j} - 2 \mathbf{k}$ and $\mathbf{B} = -\mathbf{i} - 2 \mathbf{k}$. Determine

i) $\mathbf{A} \cdot \mathbf{B}$

ii) $\mathbf{A} \times \mathbf{B}$

iii) Angle between $\mathbf{A} + \mathbf{B}$


\[ \begin{aligned} \mathbf{A} \cdot \mathbf{B} &= (3 \times 1) + (4 \times 0) + (-2 \times -2) \\ &= 3 + 4 = 7 \end{aligned} \]

\[ \begin{aligned} \mathbf{A} \times \mathbf{B} &= \begin{vmatrix} i & j & k \\ 3 & 4 & -2 \\ -1 & 0 & -2 \end{vmatrix} \\ &= (4 \cdot (-2) - (-2) \cdot 0) \mathbf{i} - (3 \cdot (-2) - (-2) \cdot 1) \mathbf{j} + (3 \cdot 0 - 4 \cdot (-1)) \mathbf{k} \\ &= (-8 + 0) \mathbf{i} - (-6 + 2) \mathbf{j} + (0 + 4) \mathbf{k} \\ &= -8 \mathbf{i} + 4 \mathbf{j} + 4 \mathbf{k} \end{aligned} \]
\[
\begin{align*}
&i(-8) - j(-8) + k(4) \\
&= -8i + 8j + 4k \\
&= -2i + 2j + k \\
&= \sqrt{4+4+1} = \sqrt{9} = 3
\end{align*}
\]

\[
\begin{align*}
|A| &= \sqrt{3^2 + 4^2 + \frac{1}{2}^2} = \sqrt{9+16+\frac{1}{4}} = \sqrt{29} \\
|B| &= \sqrt{1 + 4} = \sqrt{5}
\end{align*}
\]

\[
\begin{align*}
\theta &= \frac{\sqrt{29}}{\sqrt{10a} \sqrt{5}} \\
\theta &= 85.24^\circ
\end{align*}
\]

A cantilever beam is loaded as shown in figure. Determine the reactions.

1)

2)

3)
1) A

\[ \Sigma F_x = 0 \]
\[ R_H - 10 \cos 20^\circ = 0 \]
\[ R_H = 9.4 \text{ N.} \]

\[ \Sigma F_y = 0 \]
\[ R_V - 50 - 10 \sin 20 = 0 \]
\[ R_V = 53.42 \text{ N} \]

\[ \Sigma M = 0 \]
\[ m - (50 \times 2.5) - (10 \sin 20 \times 5) = 0 \]
\[ m = 142.10 \]

\[ R = 54.24 \text{ N} \]
\[ \theta = 90.02^\circ \]

2) A

\[ \Sigma F_y = 0 \]
\[ R_A - 35 - 10 + R_B = 0 \]
\[ R_A + R_B = 45 \]  \hspace{1cm} (1) \]
\[ \sum m_A = 0. \]
\[ 2 R_A = (3.5 \times 3.5) - (10 \times 6) + (R_B \times 6) = 0 \]
\[ 2 R_A + 6 R_B = 18.25 \quad (2) \]

\[ R_A = 21.845 \text{ N} \]
\[ R_B = 23.152 \text{ N}. \]

3) A

\[ \sum F_X = 0 \]
\[ R_{AH} - 5(0.30) = 0 \]
\[ R_{AH} = 4.33 \text{ N}. \]

\[ \sum F_Y = 0 \]
\[ R_{AV} = 15 - 15 \sin 30 + R_B = 0 \]
\[ R_{AV} + R_B = 32.5 \quad (1) \]

\[ \sum m_A = 0. \]
\[ -m - (15 \times 3.5) - (15 \times 1) - (5 \sin 30 \times 6) + (R_B \times 7) = 0 \]

\[ m = 13 \]
\[ R_B = 13.21 \text{ N} \]
\[ R_{AV} = 19.29 \text{ N} \]

\[ R = 19.77 \text{ N} \]
\[ \theta = 47.35^\circ \]
Point A (2, -1, 0) and B (1, -2, 4). Determine the unit vector in the direction of AB.

A. \[ \overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A} \]
\[ = (-1 - 2)i + (-2 + 1)j + (4 - 0)k \]
\[ = -3i - j + 4k \]

Unit vector in the given direction \( \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} \)
\[ = \frac{-3i - j + 4k}{\sqrt{9 + 1 + 16}} \]
\[ = \frac{-3i - j + 4k}{\sqrt{26}} \]
\[ = \frac{3}{\sqrt{26}}i - \frac{j}{\sqrt{26}} + \frac{4}{\sqrt{26}}k \]
\[ = -0.59i - 0.2j + 0.78k \]

Determine a force of magnitude 15 N in the direction of AB.

A. Force of magnitude 15 in the direction of AB 
\[ = 15 \times \text{unit vector in the direction of AB} \]
\[ = 15 \times (-0.59i - 0.2j + 0.78k) \]
\[ = -8.85i - 3i + 11.7k \]

Figure shows a tripod which is subjected to forces as shown. Determine the force acting on leg PB, PA, PC. Point P is 10 m above the ground.
From the figure we have coordinates of:

A (-4, 0, 0)
B (5, 0, 2)
C (-2, 0, -3)
P (0, 10, 0)

Let \( \vec{F}_PA \) is the force acting on leg PA.
\( \vec{F}_{PB} \) is the force on leg PB, \( \vec{F}_{PC} \) is the one on PC.
There are 5 forces acting on the tripod. They are

i) force in leg AP, \( \vec{F}_{AP} \)

ii) force in leg BP, \( \vec{F}_{BP} \)
iii) Force in leg CP, \( \vec{F}_{CP} \)

iv) Force of 8 kN acting horizontally at point P.
v) Force of 16 kN acting vertically at point P.

Tripot is in eqm under the influence of these five forces.

\[ \vec{F}_{AP} + \vec{F}_{BP} + \vec{F}_{CP} + 8 \text{kN} + 16 \text{kN} = \mathbf{0} \]

To find \( \vec{A}P \)

\[ \vec{A}P = \vec{P} - \vec{A} = 4\mathbf{i} + 10\mathbf{j} \]

Unit vector in direction of \( \vec{A}P \):

\[ \frac{4\mathbf{i} + 10\mathbf{j}}{\sqrt{116}} = 0.37\mathbf{i} + 0.93\mathbf{j} \]

Force acting on leg AP, \( \vec{F}_{AP} = |\vec{F}_{AP}| \times \text{unit vector} \)

\[ |\vec{F}_{AP}| \times 0.37\mathbf{i} + 0.93\mathbf{j} \quad \text{(1)} \]

\[ \vec{B}P = \vec{P} - \vec{B} = -5\mathbf{i} + 10\mathbf{j} - 2\mathbf{k} \]

Unit vector in direction of \( \vec{B}P \):

\[ \frac{-5\mathbf{i} + 10\mathbf{j} - 2\mathbf{k}}{\sqrt{129}} = -0.44\mathbf{i} + 0.98\mathbf{j} - 0.18\mathbf{k} \]

Force acting on leg BP, \( \vec{F}_{BP} = (\vec{F}_{BP}) \times 0.44\mathbf{i} + 0.98\mathbf{j} - 0.18\mathbf{k} \quad \text{(2)} \]

\[ \vec{C}P = \vec{P} - \vec{C} = 2\mathbf{i} + 10\mathbf{j} + 2\mathbf{k} \]

Unit vector in direction of \( \vec{C}P \):

\[ \frac{2\mathbf{i} + 10\mathbf{j} + 2\mathbf{k}}{\sqrt{113}} = 0.19\mathbf{i} + 0.94\mathbf{j} + 0.28\mathbf{k} \]

Force acting on leg CP, \( \vec{F}_{CP} = |\vec{F}_{CP}| \times 0.19\mathbf{i} + 0.94\mathbf{j} + 0.28\mathbf{k} \quad \text{(3)} \]
We have,
\[ \vec{F}_{AP} + \vec{F}_{BP} + \vec{F}_{CP} + 8i - 16j = 0. \]

\[ |\vec{F}_{AP}| (0.37i + 0.33j) + |\vec{F}_{BP}| (-44i + 88j - 18k) + |\vec{F}_{CP}| (19j + 94j + 28k) + 8i - 16j = 0 \]

Equating coefficients of \( i = 0 \)
\[ 0.37 |\vec{F}_{AP}| - 4.4 |\vec{F}_{BP}| + 0.19 |\vec{F}_{CP}| + 8i = 0. \] —(4)

Equate coef \( j = 0 \).
\[ 0.37 |\vec{F}_{AP}| + 88 |\vec{F}_{BP}| + 0.94 |\vec{F}_{CP}| - 16 = 0. \] —(5)

Equate coef \( k = 0 \).
\[ -18 |\vec{F}_{BP}| + 28 |\vec{F}_{CP}| = 0. \] —(6)

\[ \begin{align*}
|\vec{F}_{AP}| &= 7.685 \\
|\vec{F}_{BP}| &= 15.81 \\
|\vec{F}_{CP}| &= 10.17
\end{align*} \]

Force \( \vec{F}_{AP} = -0.44i - 2.812i - 7.068j \)
\( \vec{F}_{BP} = -6.996i + 13.992j - 2.862k \)
\( \vec{F}_{CP} = 1.932i + 9.55j + 2.87k \)

A wire is fixed at point A and B as shown in figure. Two weights 10 kN & 30 kN are supported at B & C respectively. When eqn is reached inclination of AB is 20° and that of CD is 50° to the vertical. Determine the tension in AB, BC & CD of the wire and also find the inclination of
BC to the vertical.

\[ \sum F_x = 0. \]

\[ T_{BC} \cos \theta - T_{AB} \sin 20^\circ = 0 \]

\[ T_{AB} = \frac{T_{BC} \cos \theta}{0.84} \]  \hspace{1cm} (1)

\[ T_{AB} = 2.92 T_{BC} \cos \theta \]  \hspace{1cm} (1)

\[ \sum F_y = 0. \]

\[ -T_{BC} \sin \theta + T_{AB} \cos 20^\circ + W = 0 \]

\[ T_{AB} \cos 20 = 10 + T_{BC} \sin \theta \]

\[ \cos 20 \cdot 2.92 T_{BC} \cos \theta - T_{BC} \sin \theta = 10 \]

\[ T_{BC} \left[ 2.92 \cos^2 \theta + 0.74 \cos \theta \right] = 10. \]  \hspace{1cm} (A)
\[ \sum F_y = 0. \]

\[ T_{DC} \sin 50^\circ - T_{BC} \cos \theta = 0 \]

\[ T_{DC} = 1.3 \ T_{BC} \cos \theta \] — (3)

\[ \sum F_y = 0. \]

\[ T_{DC} \cos 50^\circ + T_{BC} \sin \theta = 30 \]

\[ 1.3 T_{BC} \cos \theta \cos 50^\circ + T_{BC} \sin \theta = 30 \]

\[ T_{BC} \left[ 0.83 \cos \theta + \sin \theta \right] = 30 \] — (B)

\[ \frac{B}{A} = \frac{0.83 \cos \theta + \sin \theta}{2.14 \cos \theta \ - \ sin \theta} = 2 \]

\[ 18 \cos \theta + \sin \theta = 8 \cdot 22 \cos \theta - 3 \sin \theta \]

\[ 4 \sin \theta = 7.39 \cos \theta \]

\[ \frac{\sin \theta}{\cos \theta} = \frac{7.39}{4} \]

\[ \tan \theta = \frac{7.39}{4} \]

\[ \theta = 61.57^\circ \]
\[ T_{bc} = 23.52 \text{ kN} \]
\[ T_{co} = 14.56 \text{ kN} \]
\[ T_{AB} = 328.64 \text{ kN} \]

Two cylinders are placed in a trough as shown in Fig. neglecting friction and the reaction at all contact surfaces. Given dia of 1st cylinder is 120 mm, dia of II cylinder is 60 mm, wt of I cylinder 250N & wt of II cylinder 100N.

\[ \alpha = \sin^{-1} \left( \frac{30}{90} \right) = 33.75^\circ \]

\[ \begin{align*}
\sum F_x &= 0. \\
R_2 \sin 33.7 + R_4 \cos 45 - R_3 &= 0. \quad (1) \\
\sum F_y &= 0. \\
- R_2 \cos 33.7 + R_4 \sin 45 - W &= 0. \\
R_4 \sin 45 - R_2 \cos 33.7 &= 250. \quad (2)
\end{align*} \]
A tripod supports a load of 8kN as shown in the figure. The ends A, B, and C are in the x-z plane. Find all the forces acting on the legs of the tripod.
We have coordinates of
A (1.2, 0, 0)
B (0, 0, 1.2)
c (-1, 0, -0.8)
D (0, 1.8, 0)

\[ \mathbf{AD} = -1.2 \mathbf{i} + 1.8 \mathbf{j} \]

Unit vector in direction of \( \mathbf{AD} \) is

\[ \mathbf{\hat{AD}} = \frac{1.2 \mathbf{i} + 1.8 \mathbf{j}}{\sqrt{1.2^2 + 1.8^2}} \]

\[ = 0.68 \mathbf{i} + 0.88 \mathbf{j} \]

force acting on leg \( \mathbf{AD} \), \( \mathbf{F_{AD}} = |\mathbf{F_{AD}}| (0.68 \mathbf{i} + 0.88 \mathbf{j}) \) --- (1)

\[ \mathbf{BD} = 0.8 \mathbf{j} - 1.2 \mathbf{k} \]

Unit vector in direction of \( \mathbf{BD} \) is

\[ \mathbf{\hat{BD}} = \frac{0.8 \mathbf{j} - 1.2 \mathbf{k}}{2.16} \]

\[ = 0.36 \mathbf{j} - 0.56 \mathbf{k} \]

force acting on leg \( \mathbf{BD} \), \( \mathbf{F_{BD}} = |\mathbf{F_{BD}}| (0.36 \mathbf{j} - 0.56 \mathbf{k}) \) --- (2)

\[ \mathbf{CD} = 1 \mathbf{i} + 1.8 \mathbf{j} + 0.8 \mathbf{k} \]

Unit vector in direction of \( \mathbf{CD} \) is

\[ \mathbf{\hat{CD}} = \frac{1 \mathbf{i} + 1.8 \mathbf{j} + 0.8 \mathbf{k}}{2.21} \]

\[ = 0.45 \mathbf{i} + 0.81 \mathbf{j} + 0.36 \mathbf{k} \]

force acting on leg \( \mathbf{CD} \), \( \mathbf{F_{CD}} = |\mathbf{F_{CD}}| (0.45 \mathbf{i} + 0.81 \mathbf{j} + 0.36 \mathbf{k}) \) --- (3)
We have:

\[
\vec{F}_{AD} + \vec{F}_{BD} + \vec{F}_{CD} + 2\vec{j} = 0.
\]

\[
|\vec{F}_{AD}| (0.56i + 0.83j) + |\vec{F}_{BD}| (0.83j - 0.56k) + |\vec{F}_{CD}| (0.45i + 0.81j + 0.36k) + 2\vec{j} = 0.
\]

\[
\text{coeff } i = 0:
\]

\[
-0.56|\vec{F}_{AD}| + 0.45|\vec{F}_{CD}| = 0.
\]

\[
j = 0:
\]

\[
0.83|\vec{F}_{AD}| + 0.83|\vec{F}_{BD}| + 0.81|\vec{F}_{CD}| + 2 = 0.
\]

\[
k = 0:
\]

\[
-0.56|\vec{F}_{BD}| + 0.36|\vec{F}_{CD}| = 0.
\]

\[
|\vec{F}_{AD}| = 0.799 \text{ kN}
\]

\[
|\vec{F}_{BD}| = 0.639 \text{ kN}
\]

\[
|\vec{F}_{CD}| = 0.995 \text{ kN}
\]

\[
\text{Forces: } \vec{F}_{AD} = -0.45i + 0.66j.
\]

\[
\vec{F}_{BD} = 0.53j - 0.36k.
\]

\[
\vec{F}_{CD} = 0.45i + 0.805j + 0.36k.
\]
A post is held in vertical position by three cables AB, AC, AD as shown in figure. If the tension in the cable AB is 40 kN, calculate the required tension in AC + AD so that the resultant of three forces at A is vertical.

We have co-ordinates of:
A (0, 48, 0)
B (16, 0, 12)
C (0, 0, 26)
D (-14, 0, 0)

\[
\vec{AB} = -16\hat{i} + 48\hat{j} - 12\hat{k},
\]

Unit vector in direction of \(\vec{BA} = \frac{-16\hat{i} + 48\hat{j} - 12\hat{k}}{52} = -0.31\hat{i} + 0.92\hat{j} - 0.23\hat{k}\)

force acting on leg BA, \(\vec{BD} = |\vec{BD}| \cdot 0.31\hat{i} + 0.92\hat{j} - 0.23\hat{k}\)

\[
= 40 [0.31\hat{i} + 0.92\hat{j} - 0.23\hat{k}] = -12\hat{i} + 36\hat{j} - 8\hat{k}.
\]
\[ \overrightarrow{CA} = -16i + 48j + 24k. \]

Unit vector in direction of \( \overrightarrow{CA} = \frac{-16i + 48j + 24k}{56} \)
\[ = -0.28i + 0.86j + 0.43k \]

\[ \mathbf{f}_{CA} = |\mathbf{f}_{CA}| \begin{bmatrix} -0.28 \ i + 0.86 \ j + 0.43 \ k \end{bmatrix} \]  \[ \text{(1)} \]

\[ \overrightarrow{DA} = 14i + 48j \]

Unit vector in direction of \( \overrightarrow{DA} = \frac{14i + 48j}{50} \)
\[ = 0.28i + 0.96j \]

\[ \mathbf{f}_{DA} = |\mathbf{f}_{DA}| \begin{bmatrix} 0.28 \ i + 0.96 \ j \end{bmatrix} \]  \[ \text{(2)} \]

We have,
\[ \mathbf{f}_{BA} + \mathbf{f}_{CA} + \mathbf{f}_{DA} = 0 \]
\[ (-12i + 36j - 8k) + |\mathbf{f}_{CA}|(-0.28i + 0.86j + 0.43k) + |\mathbf{f}_{DA}|(0.28i + 0.96j) = 0. \]

Equating \( i = 0 \):
\[ -12 - 0.28|\mathbf{f}_{CA}| + 0.28|\mathbf{f}_{DA}| = 0. \]

Equating \( k = 0 \):
\[ -8 + 0.43|\mathbf{f}_{CA}| = 0 \]
\[ \text{we cannot equate coeff. of j to zero; hence given resultant is vertical.} \]
\[ |\mathbf{f}_{CA}| = 18.604 \text{ kN} \]
\[ |\mathbf{f}_{DA}| = 60.46 \text{ kN} \]
36 + .86 \( \begin{pmatrix} F_{EA} \\ F_{ED} \end{pmatrix} \) + .96 \( \begin{pmatrix} F_{ED} \\ F_{EC} \end{pmatrix} \) = C

C = 111.001\text{KN}

Resultant of three forces = 110.364\text{KN}